Ensemble forecasting in coupled fast/slow systems

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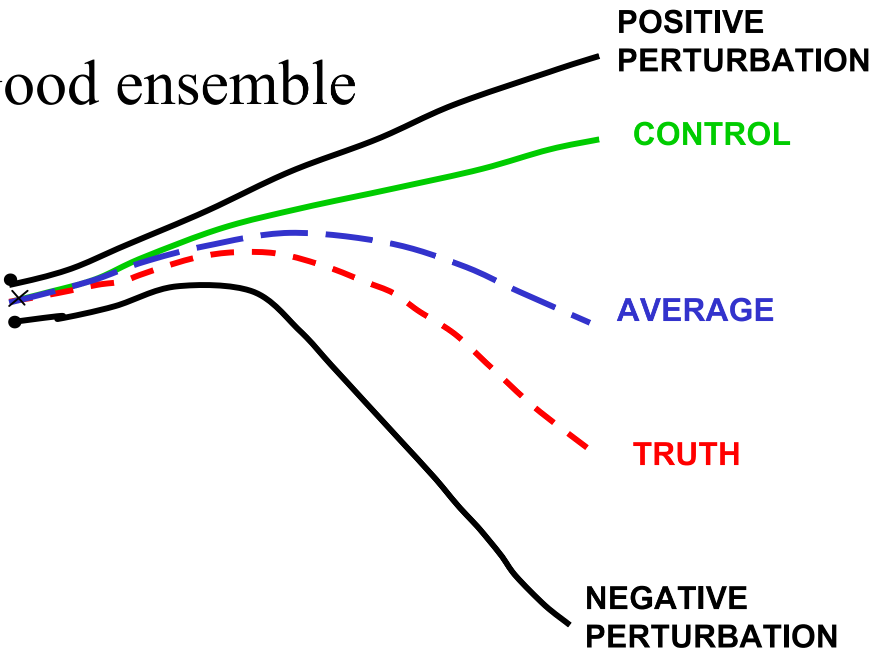
(thanks to Jim Hansen for the coupled Lorenz model code)

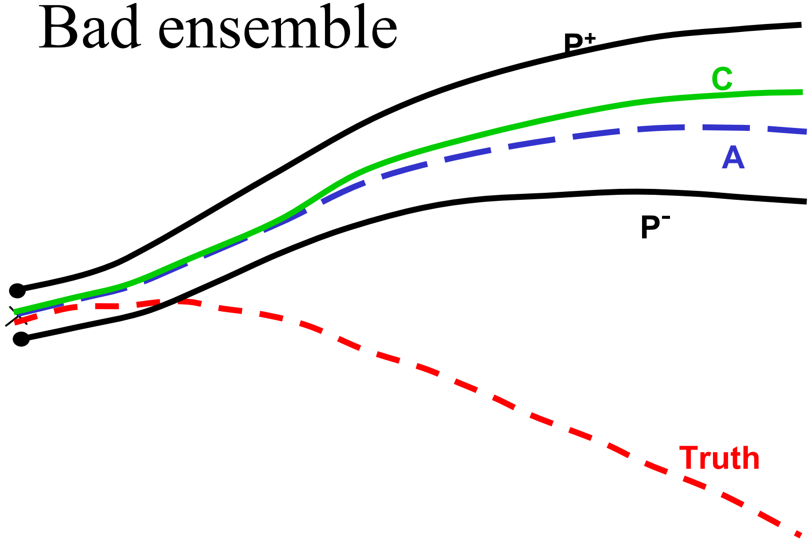
- The atmosphere has coupled instabilities that span many scales, from ENSO to brownian motion:
- ENSO has a doubling time of about one month
- Baroclinic weather waves – 2 days doubling time
- Mesoscale phenomena – a few hours
- Cumulus convection – 10 minutes
- Brownian motion – …
- Linear approaches, like Lyapunov and Singular Vectors *can only handle the fastest growing instability present in the model, nonlinear integrations allow fast instabilities to saturate*
- This has major implications for ensemble forecasting and data assimilation…

- A good ensemble should *contain the relevant unstable perturbations:* For example, an ensemble for seasonal prediction should have initial perturbations that contain coupled instabilities.

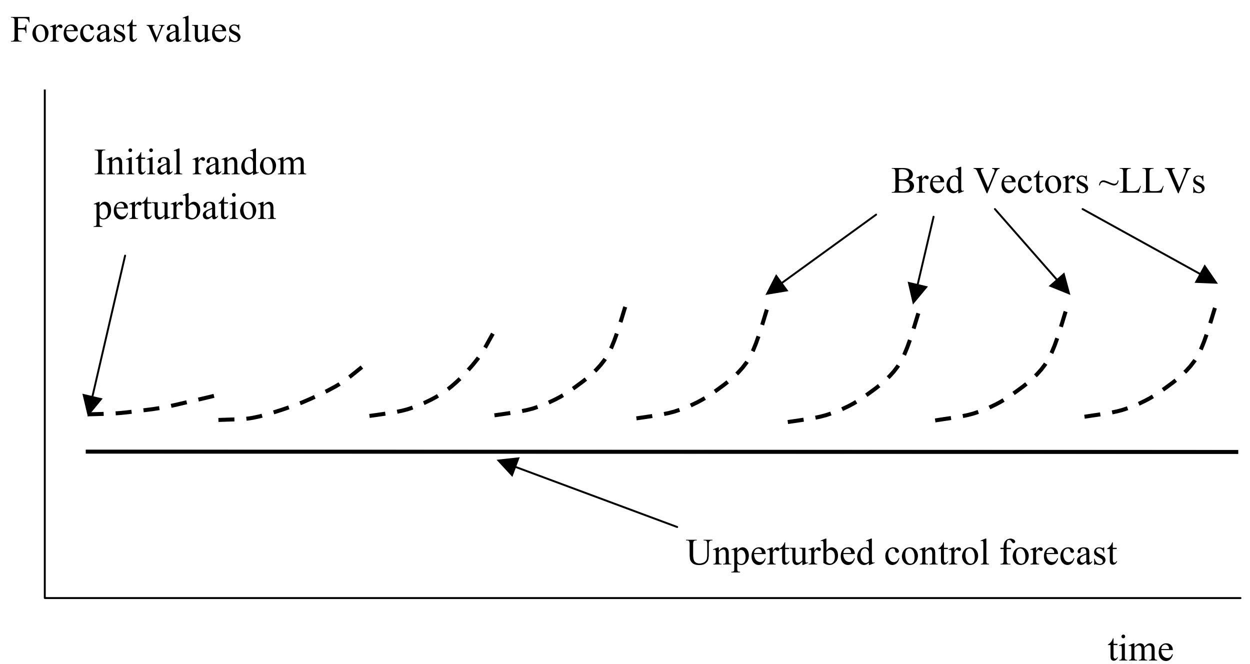
Good ensemble



Bad ensemble



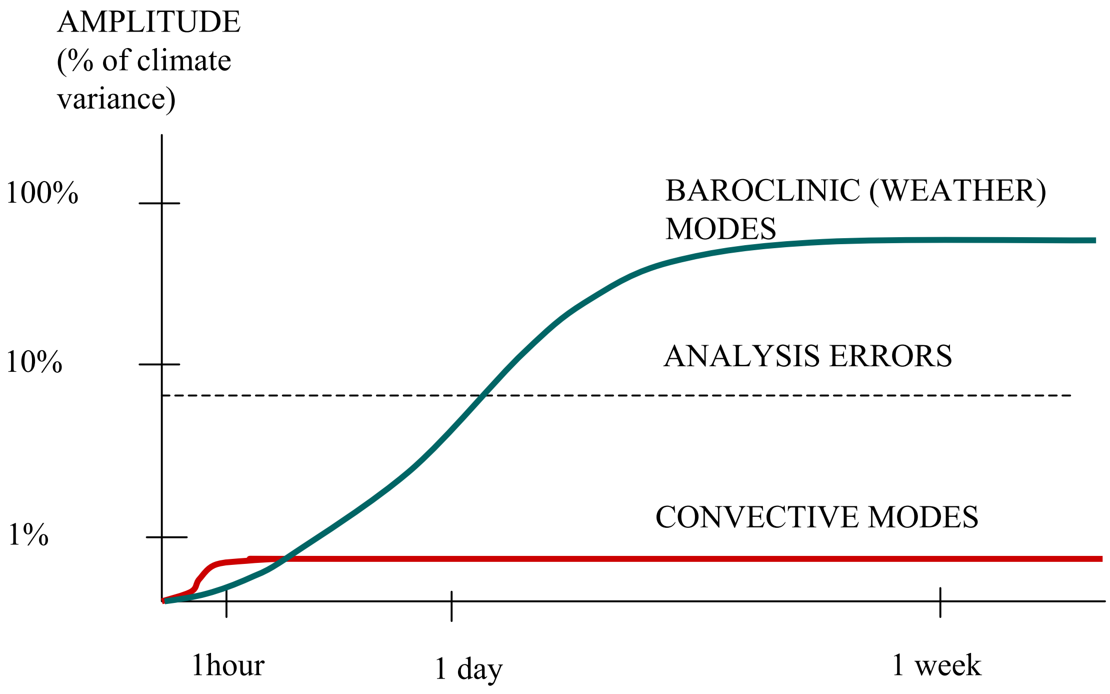
To help understand the problem in coupled systems, test breeding, simply running the nonlinear model a second time, from perturbed initial conditions. The results should be valid for EnKF and other nonlinear approaches.



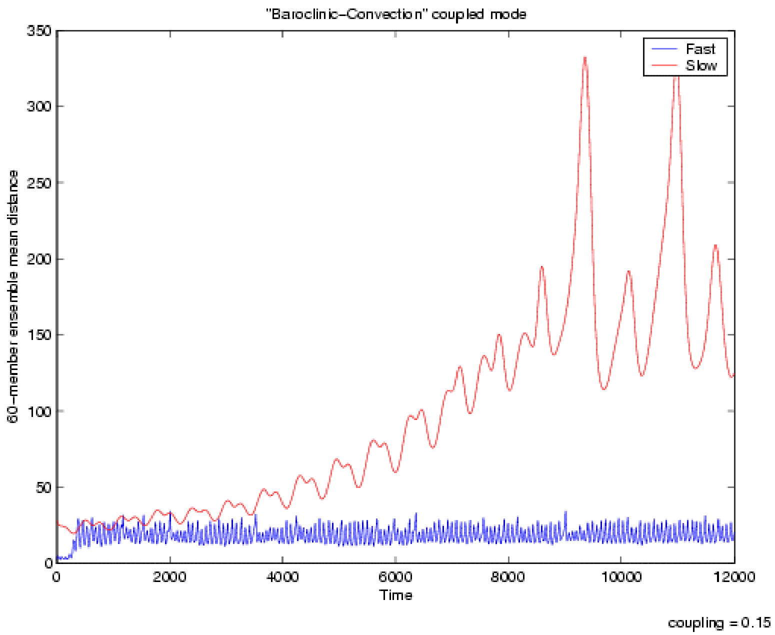
The local breeding growth rate is given by

$$g(t) = \frac{1}{n\Delta t} \ln\left(|\delta \mathbf{x}| / |\delta \mathbf{x}_0|\right)$$

In coupled systems nonlinear saturation allows filtering unwanted fast, small amplitude, growing instabilities like convection (Toth and Kalnay, 1993)



"weather + convection" coupled model



This was also noted by Aurell et al (1996, 1997) who defined a Finite Size Lyapunov Exponent (FSLE):

$$\lambda(\delta) = \frac{1}{\langle T_r(\delta) \rangle} \ln r, \qquad r = \sqrt{2} \text{ or } 2$$

The FSLE is clearly related to the Average Bred Growth Rate

$$ABGR = \left\langle \frac{1}{n\Delta t} \ln\left(|\delta \mathbf{x}| / |\delta \mathbf{x}_0| \right) \right\rangle$$

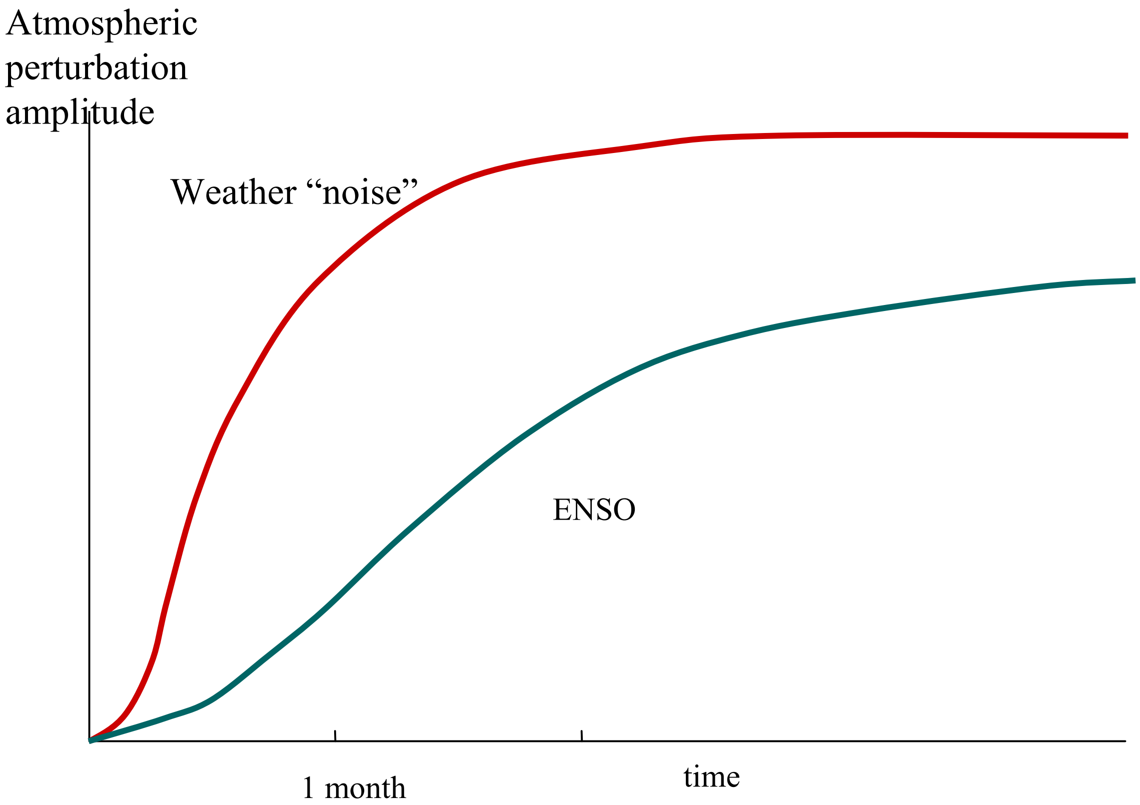
ABGR is easier to compute than FSLE, but they both converge to the Lyapunov exponent for infinitesimal amplitudes and intervals

Boffetta et al (1998) used a coupled fast/slow Lorenz model to show that the predictability is a function of the tolerance Δ, and for small amplitude fast modes, predictability is larger than the Lyapunov estimation

$$T_p = \int_{\delta}^{\Delta} \frac{d \ln \delta'}{\lambda(\delta')} \approx \frac{1}{\lambda_{\max}} \ln\left(\frac{\Delta}{\delta}\right)$$

$$T_p = \frac{1}{\lambda} \ln\left(\frac{\Delta}{\delta}\right)$$

In the case of coupled ocean-atmosphere modes, we cannot take advantage of the small amplitude of the “weather noise”! We can only use the fact that the coupled ocean modes are slower…



Test with a simple model: Lorenz (1963) 3-variable model

it has two regimes, and the transition between them is chaotic

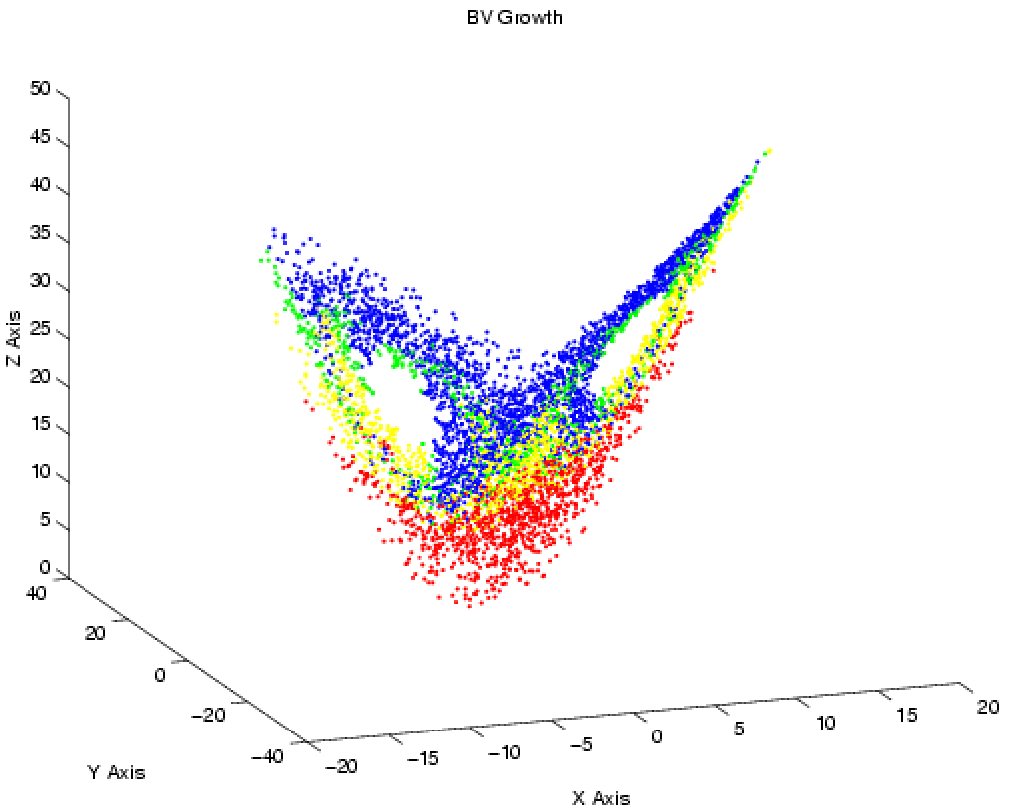
$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

In the 3-variable Lorenz (1963) model we used breeding to estimate the local growth of perturbations:

Bred Vector Growth: red, high growth; yellow, medium; green, low growth; blue, decay

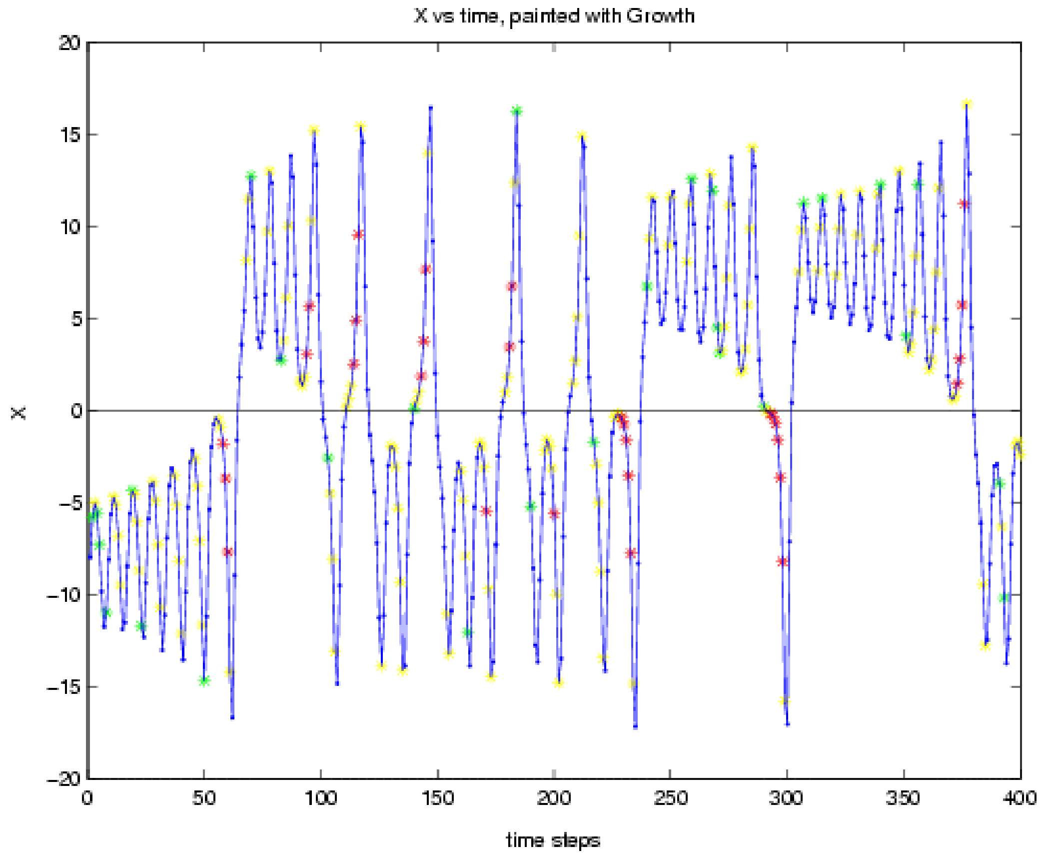


With just a single breeding cycle, we can estimate the stability of the attractor (Evans et al, 2003)

Rules for a forecaster living in the Lorenz attractor (Evans et al, 2003)

"Warm"

"Cold"



BV Growth

over 8 steps:

* BVG>1.8

* BVG>1.1

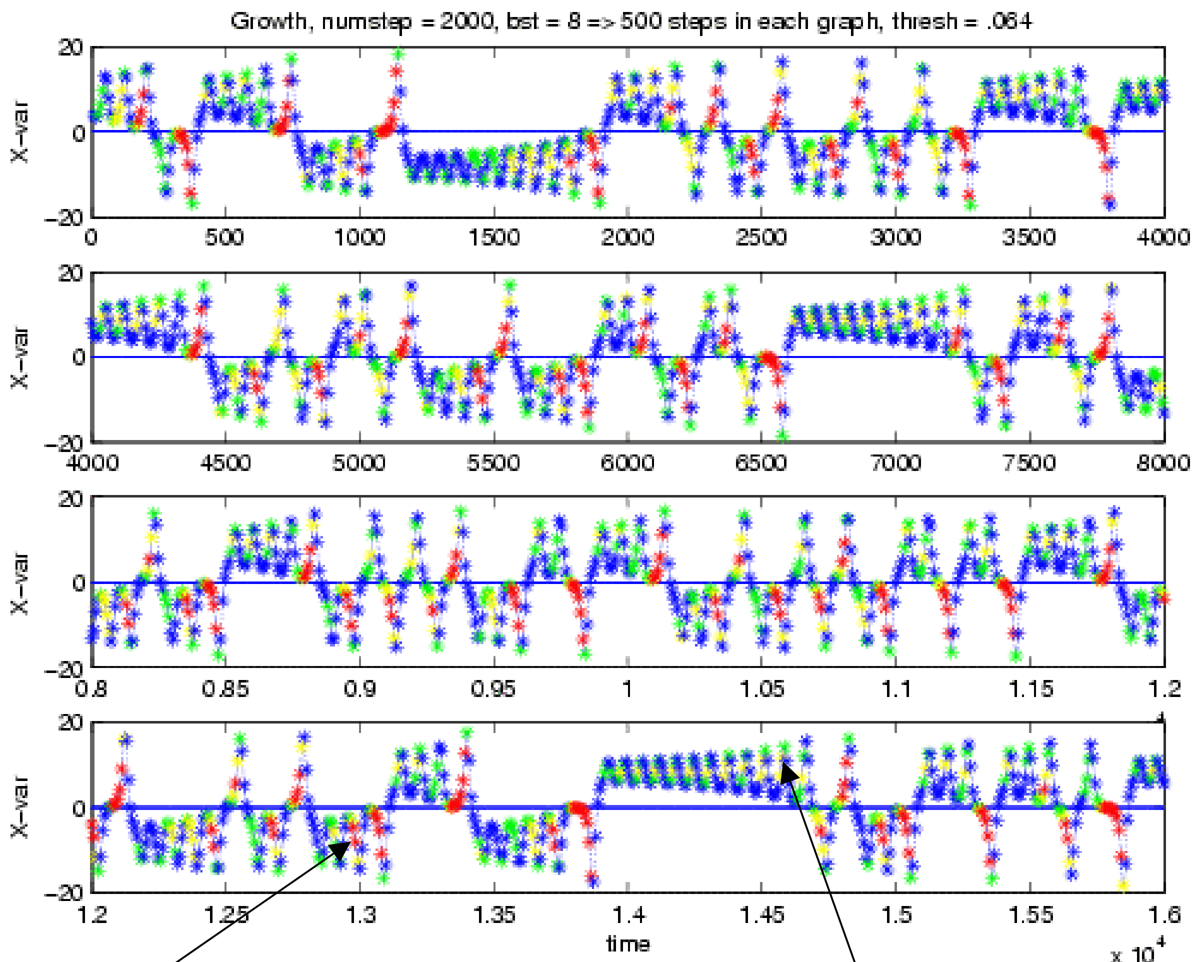
*BVG>1.0

•BVG<1.0

1) Regime change: The presence of red stars indicates that the next orbit will be the last one in the present regime.

2) Regime duration: One or two red stars, next regime will be short. Several red stars: the next regime will be long lasting.

The two rules are very robust, with threat scores >90%



Occasional false alarm

Occasional missed regime change

Breeding in a coupled system

- Breeding: finite-amplitude, finite-time instabilities of the system (~Lyapunov vectors)
- In a coupled system there are fast and slow modes, and a **linear** Lyapunov approach will only capture fast modes.
- Can we do breeding of the slow modes?

We coupled a slow and a fast Lorenz (1963) 3-variable model (Pena and Kalnay, 2003)

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1(Sz_2)$$

Slow equations

$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$

$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2(z_1)$$

We consider three representative cases:

a) “Weather waves coupled with convection” in which the fast waves are 10 times smaller and 10 times faster, and the coupling is weak:

$$\tau = 0.1, S = 0.1, C_1 = C_2 = 0.15$$

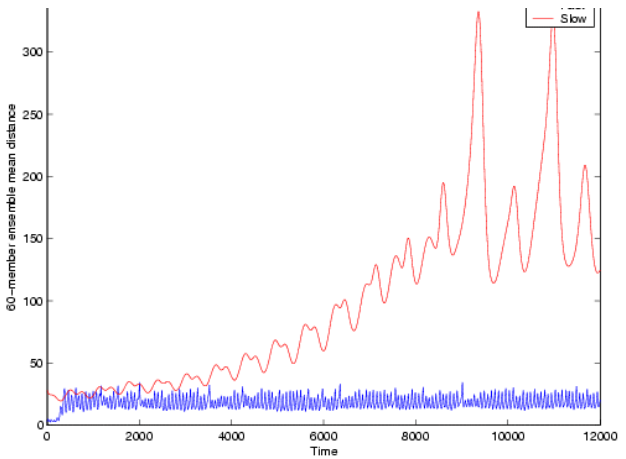
b) “ENSO coupled ocean-atmosphere” in which the fast waves have the same space scale, but are 10 times faster. We also used stronger coupling:

$$\tau = 0.1; S = 1.0; C_1 = C_2 = 1.0 ; O = -11$$

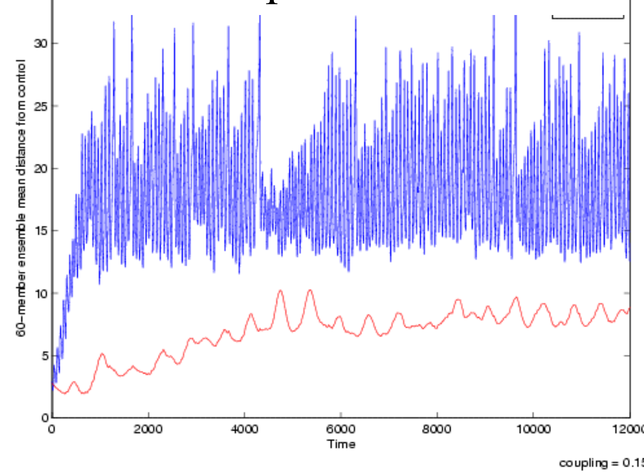
c) “Tropical-extratropical” system in which the ENSO tropical atmosphere is weakly coupled to a fast extratropical “atmosphere” (triply-coupled)

First consider the rms ensemble distances of several coupled systems

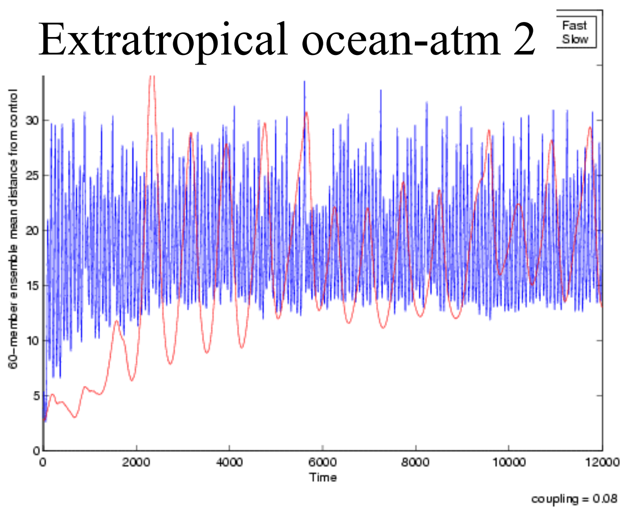
Weather wave with convection



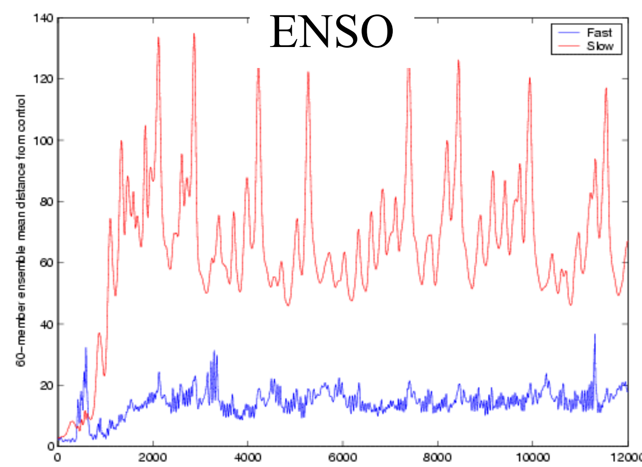
Extratropical ocean-atm 1



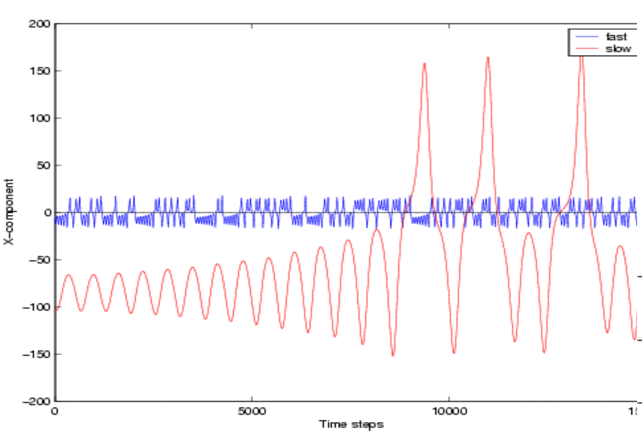
Extratropical ocean-atm 2



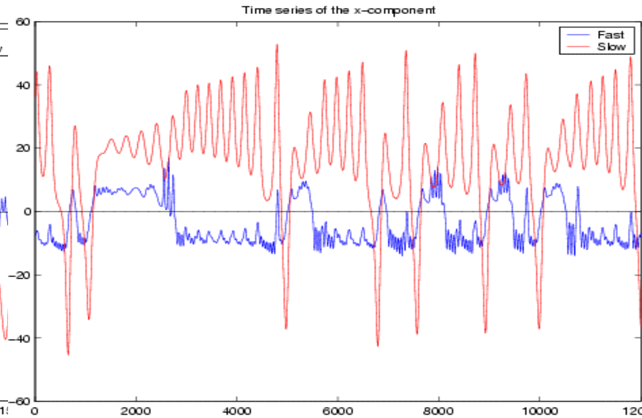
ENSO



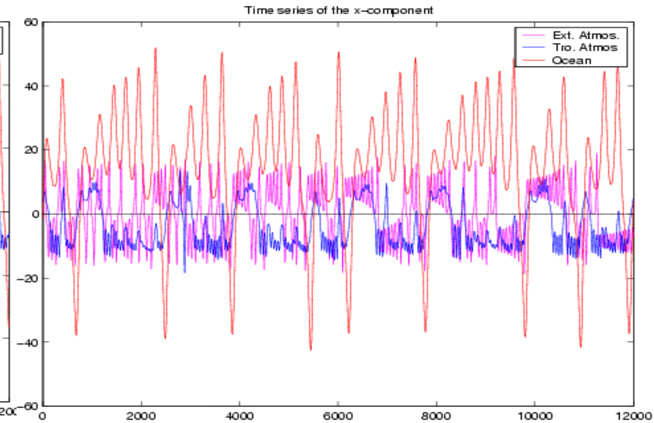
x-component of the three coupled systems



Weather waves with convection



ENSO

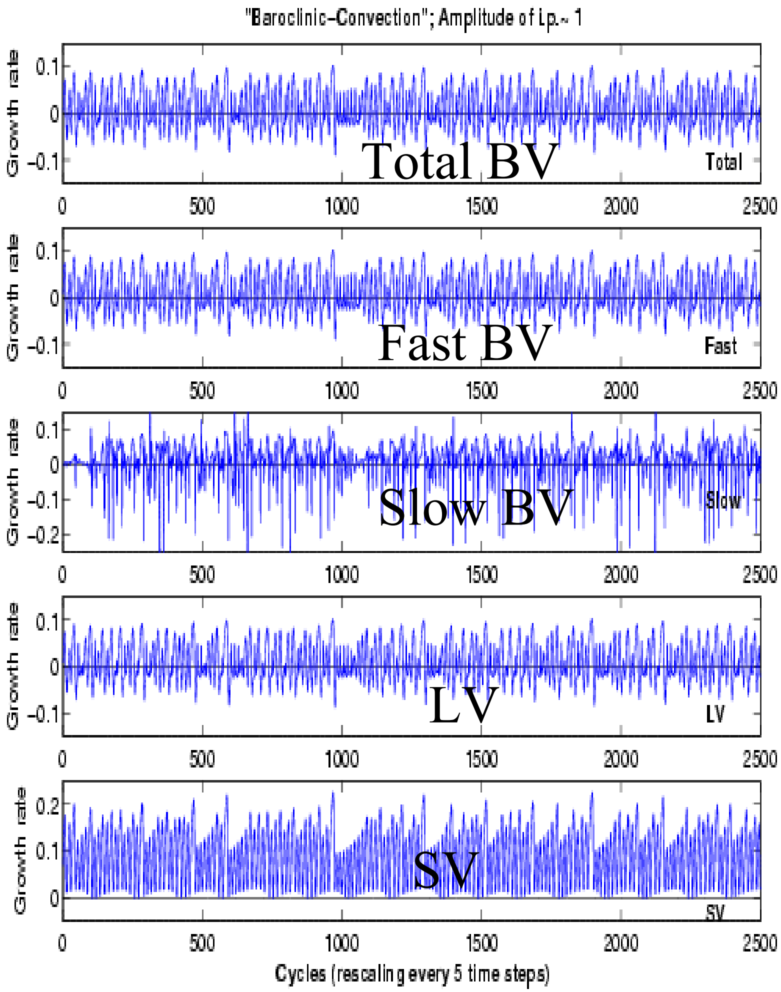


ENSO coupled to an extratropical atmosphere

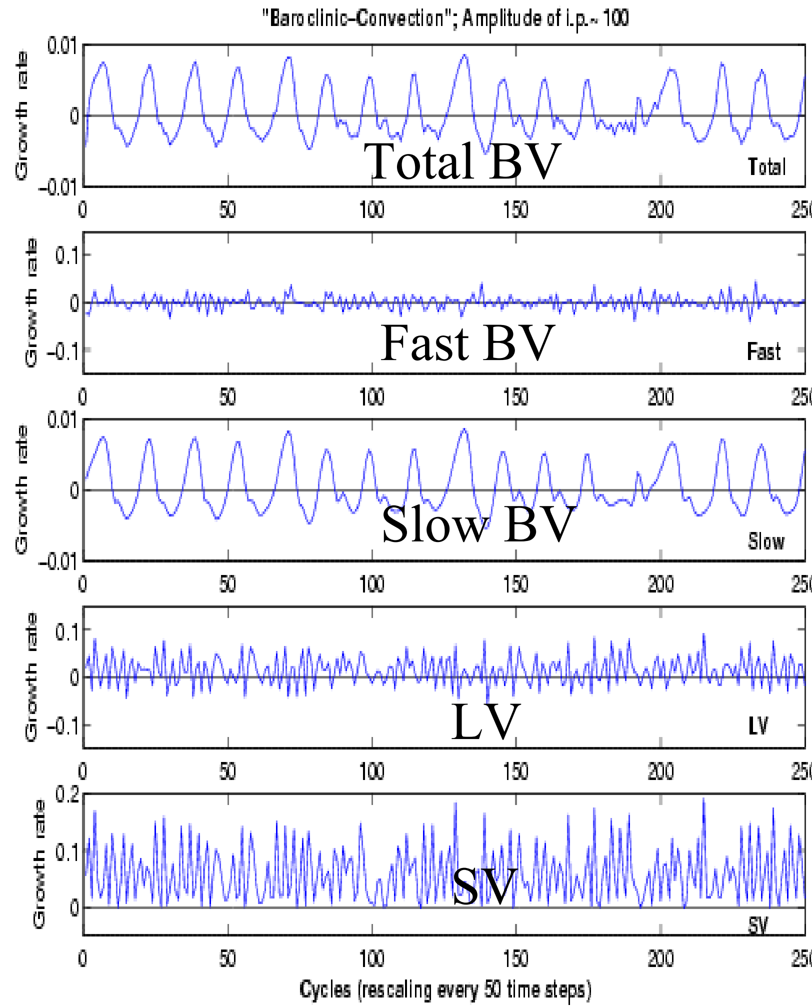
- We will use the amplitude and rescaling interval of breeding to try to separate fast and slow modes

•Compare results with those obtained using Lyapunov vectors and Singular Vectors (with the same optimization interval)

“Weather waves with convection”: we can get either the BVs for fast “convection”or slow “weather waves”, depending on the rescaling



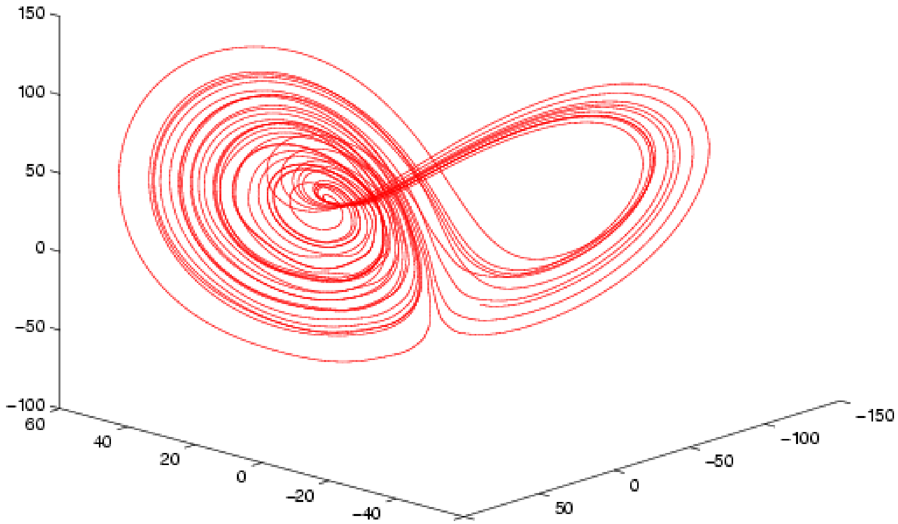
$\delta = 1, \Delta = 5$ steps



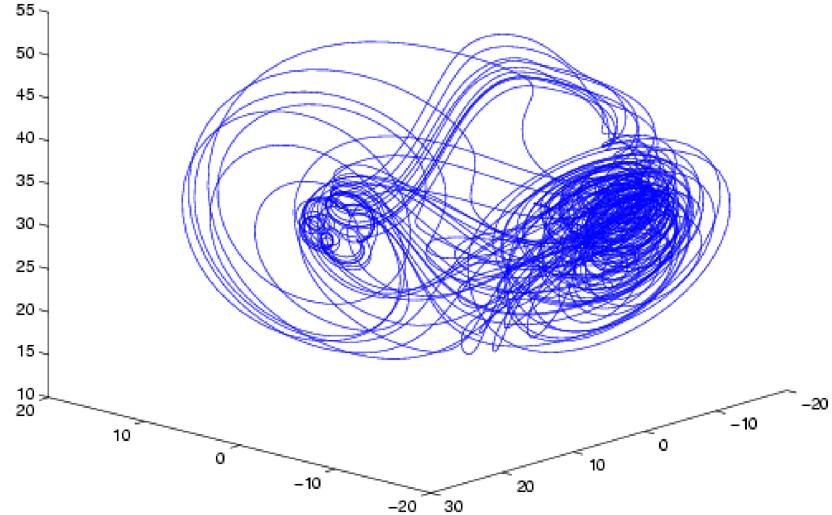
$\delta = 100, \Delta = 50$ steps

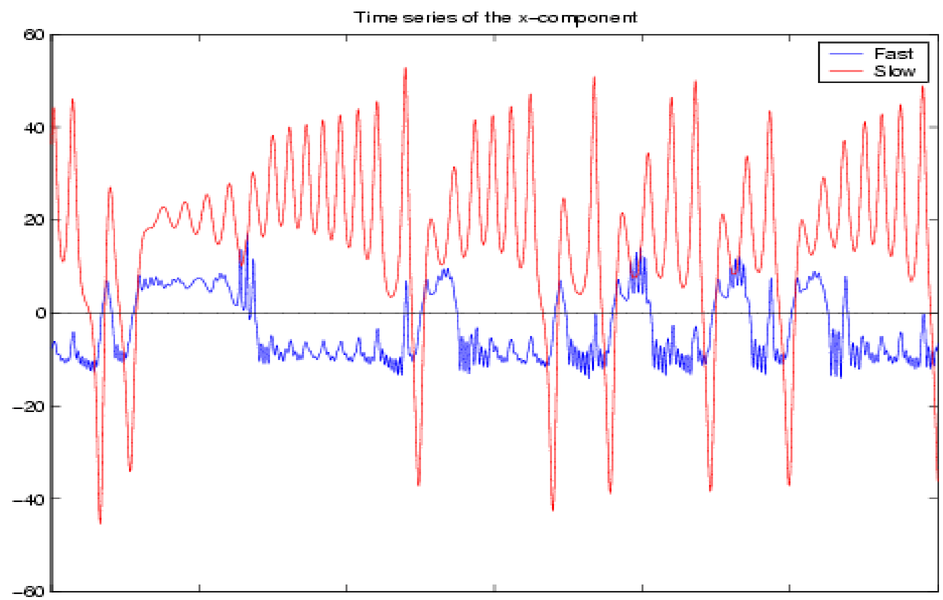
“ENSO” strongly coupled system: almost slave atmosphere

“slow ocean”

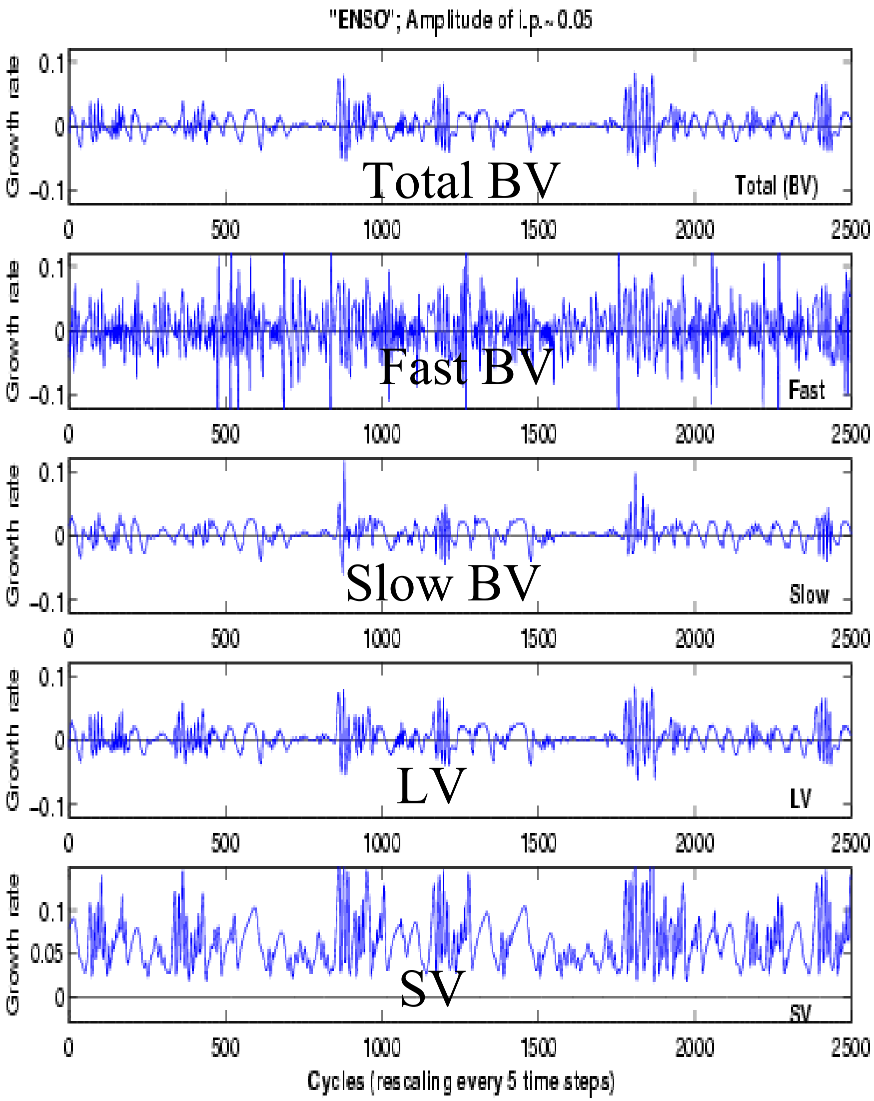


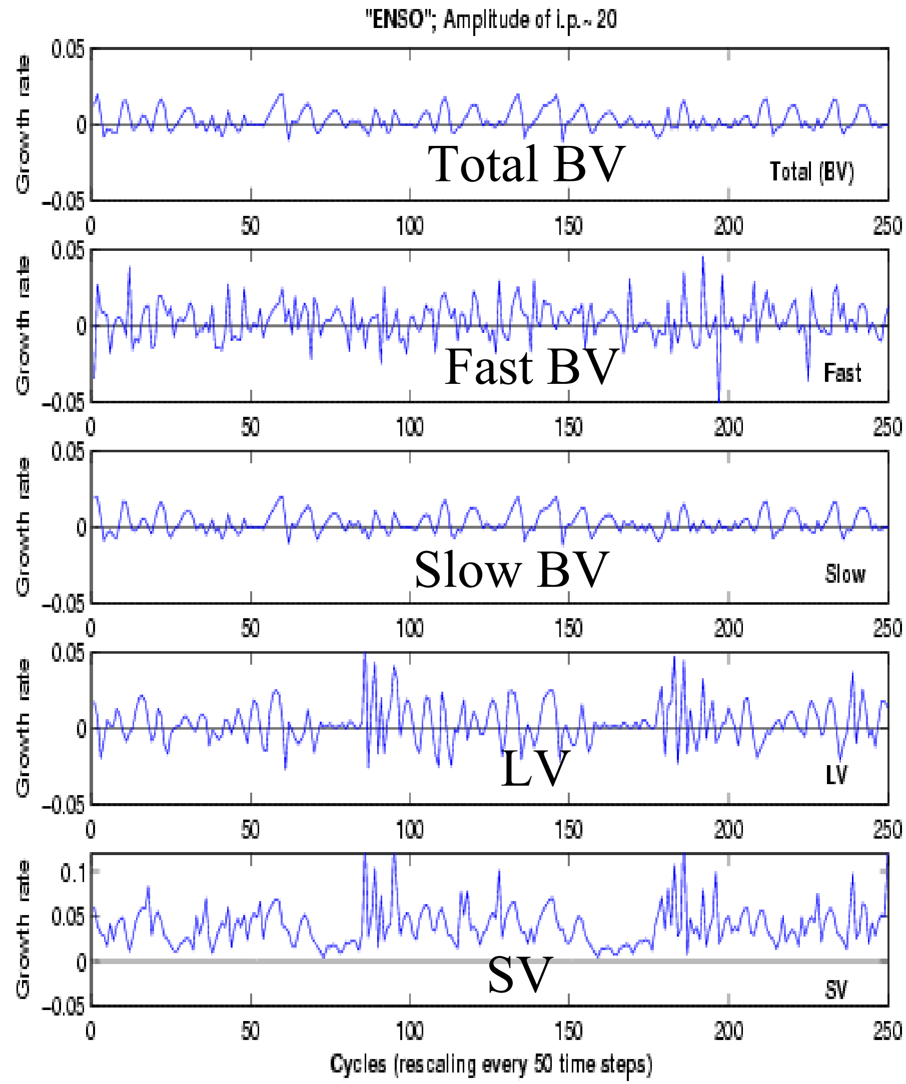
“fast atmosphere”





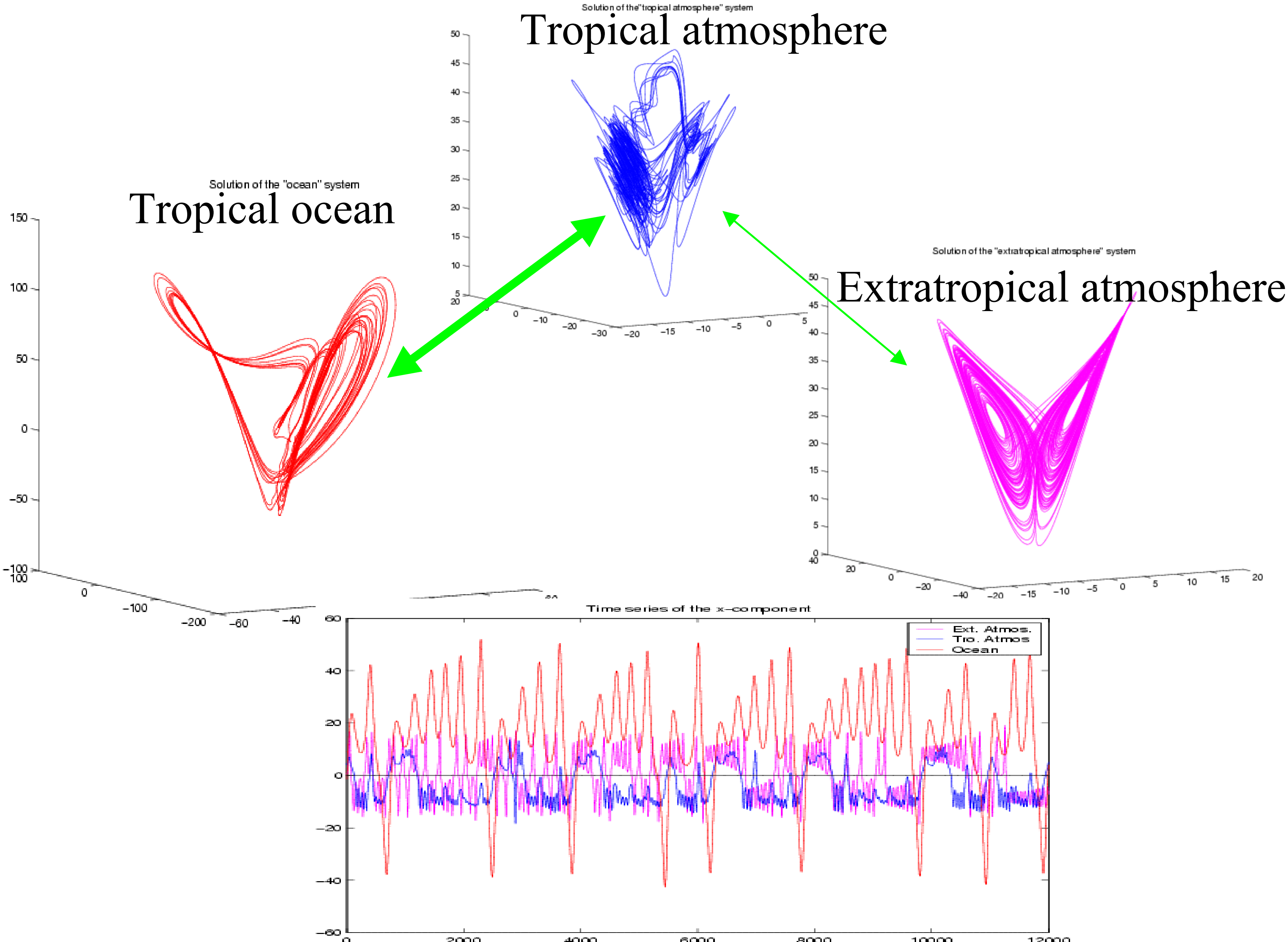
For the ENSO system we can get the BVs for slow "coupled tropics" or the fast "atmosphere" (almost a "slave")





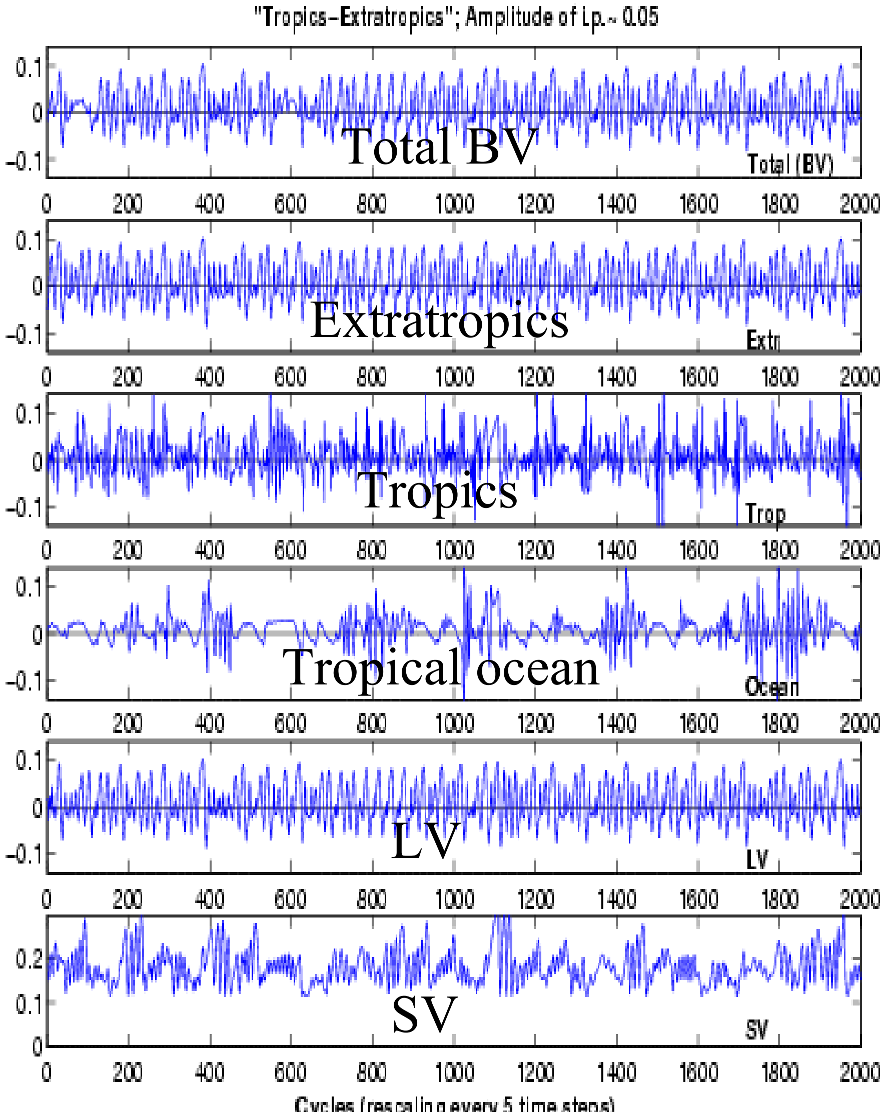
$\delta = 1, \Delta = 5$ steps

$\delta = 20, \Delta = 50$ steps

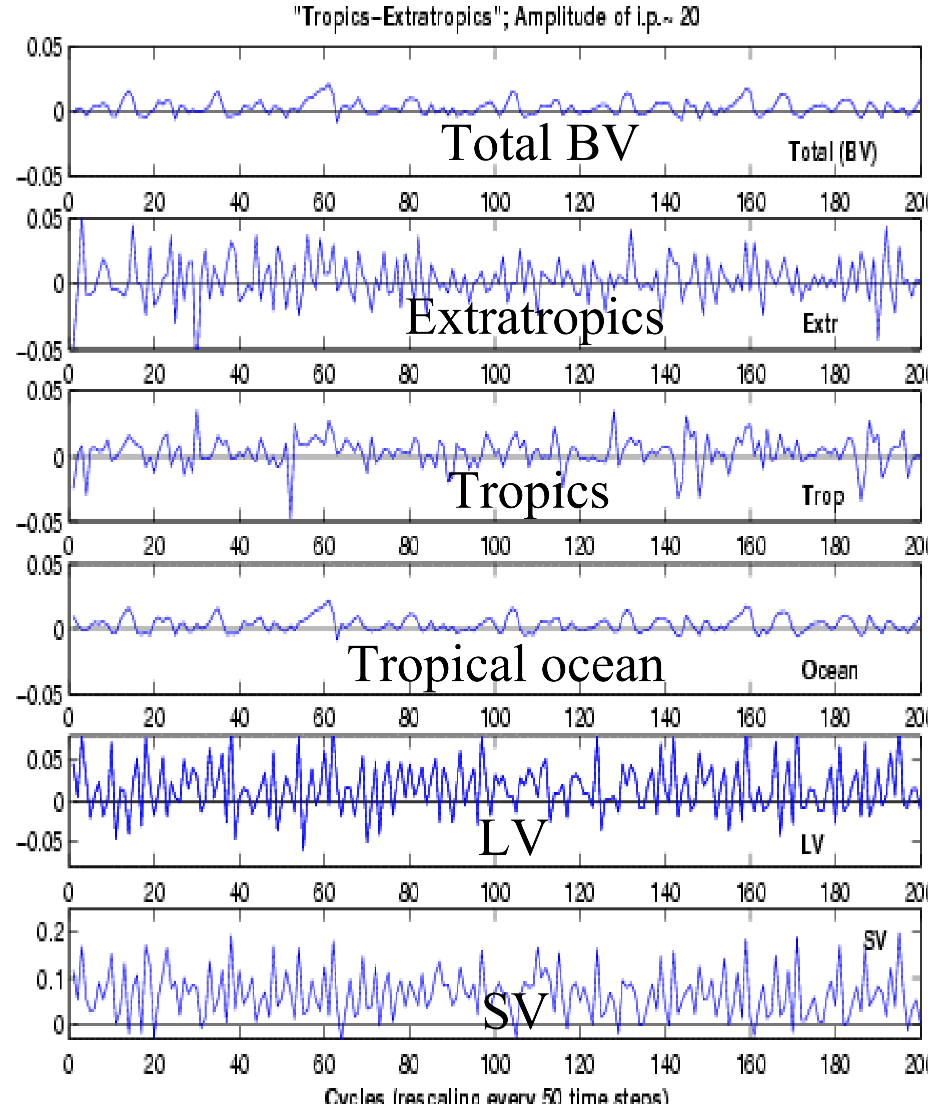


Solution of the"tropical atmosphere" system
Tropical atmosphere
Solution of the "ocean" system
Tropical ocean
Solution of the "extratropical atmosphere" system
Extratropical atmosphere
Time series of the x–component
Ext. Atmos.
Tro. Atmos
Ocean

Triply coupled system: extratropical atmosphere - ENSO



$\delta = 0.05, \Delta = 5$ steps

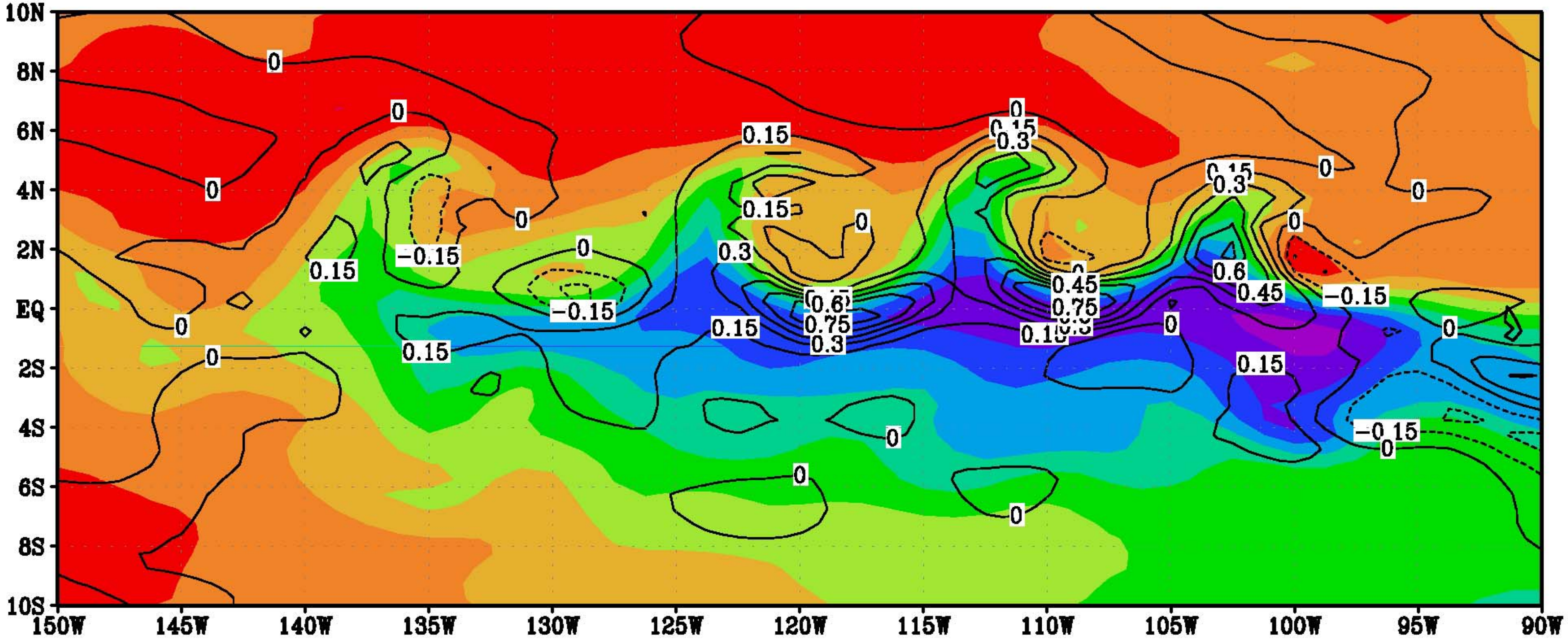


$\delta = 20, \Delta = 50$ steps

Results from Lorenz coupled models

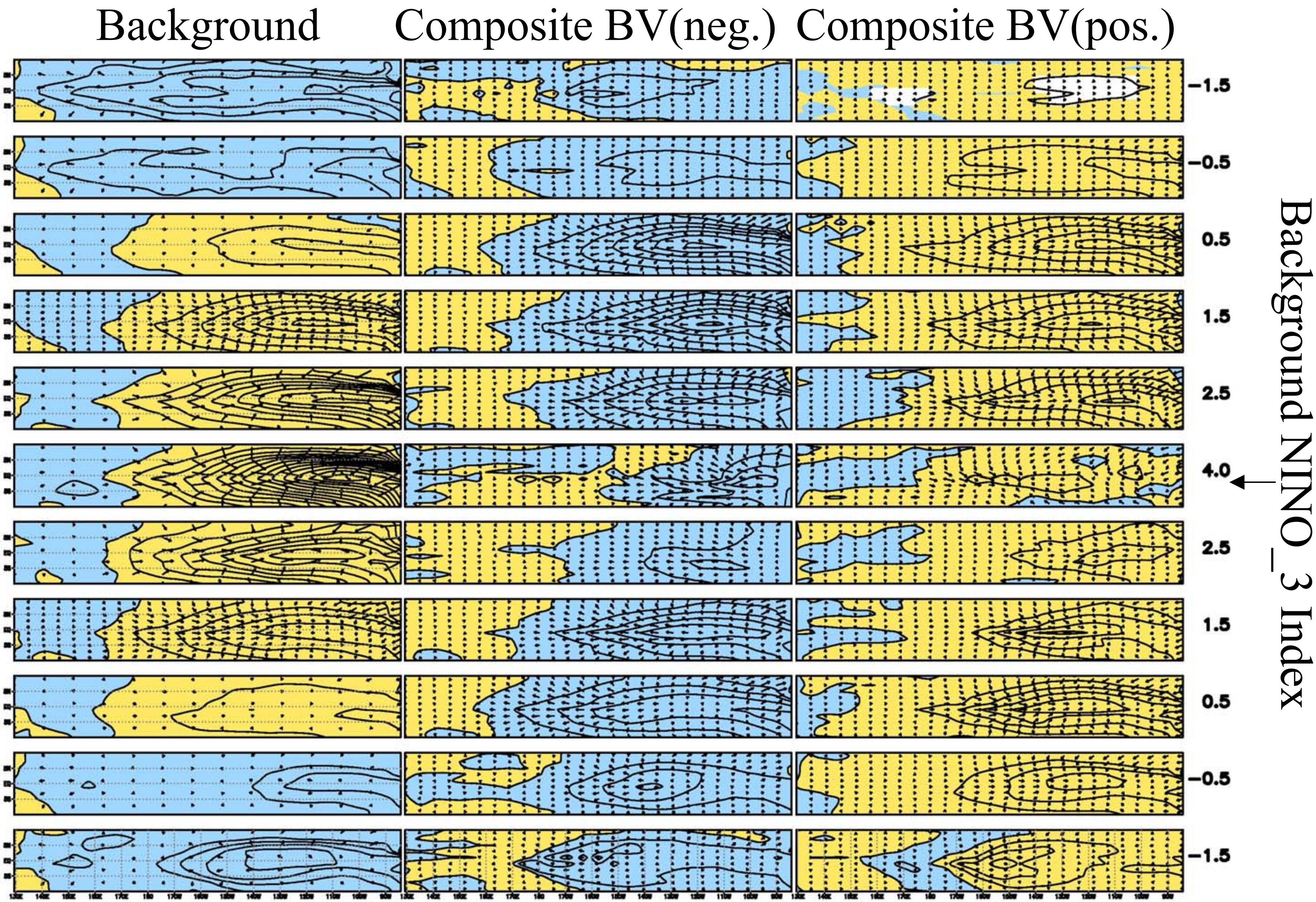
- Coupling a fast and a slow Lorenz model, we can do breeding of the slow modes
- Valid for other nonlinear approaches (e.g., EnKF) but not of linear apporaches (e.g., LVs and SVs) which are dominated by the fastest component
- Can be applied to the ENSO coupled instabilities (Cai, Kalnay and Toth, 2002, for the Zebiak-Cane model)
- We have also had promising results with the NASA NSIPP coupled ocean-atmosphere GCM (Yang, Cai and Kalnay, 2003)

In the NASA coupled GCM, there are also equatorial unstable waves in the equatorial cold tongue (color). The bred vectors (contours) give the most unstable perturbations. This provides a powerful tool for a dynamical analysis. (Yang et al, 2003)



Experiments with coupled systems

1. Zebiak-Cane model (Cai et al, 2002, J of Cl):
   - We found the instabilities of the ENSO evolution, and their dependence on the annual cycle and the ENSO phase
   - We tested the impact of bred vector ensembles and of minimizing the projection on bred vectors in the initial conditions
2. NSIPP coupled GCM
   - We performed two independent breeding experiments
   - Encouraging results suggest we can isolate the ENSO instabilities
3. Breeding with the NSIPP operational system
   - Underway

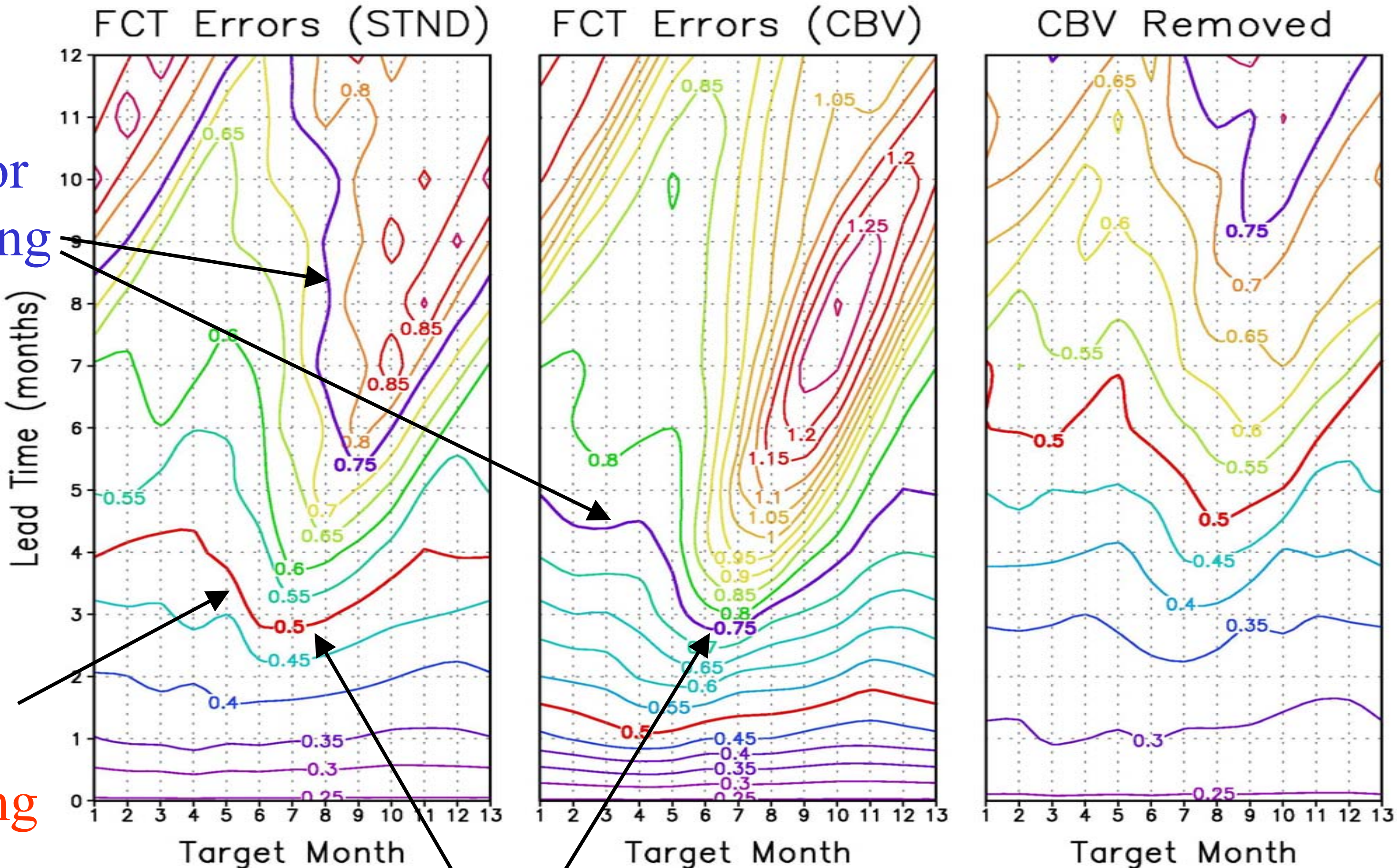


Background
Composite BV(neg.)
Composite BV(pos.)
−1.5
−0.5
0.5
1.5
2.5
4.0
2.5
1.5
0.5
−0.5
−1.5
Background NINO_3 Index

Forecast Error Growth

Error tripling

Error doubling



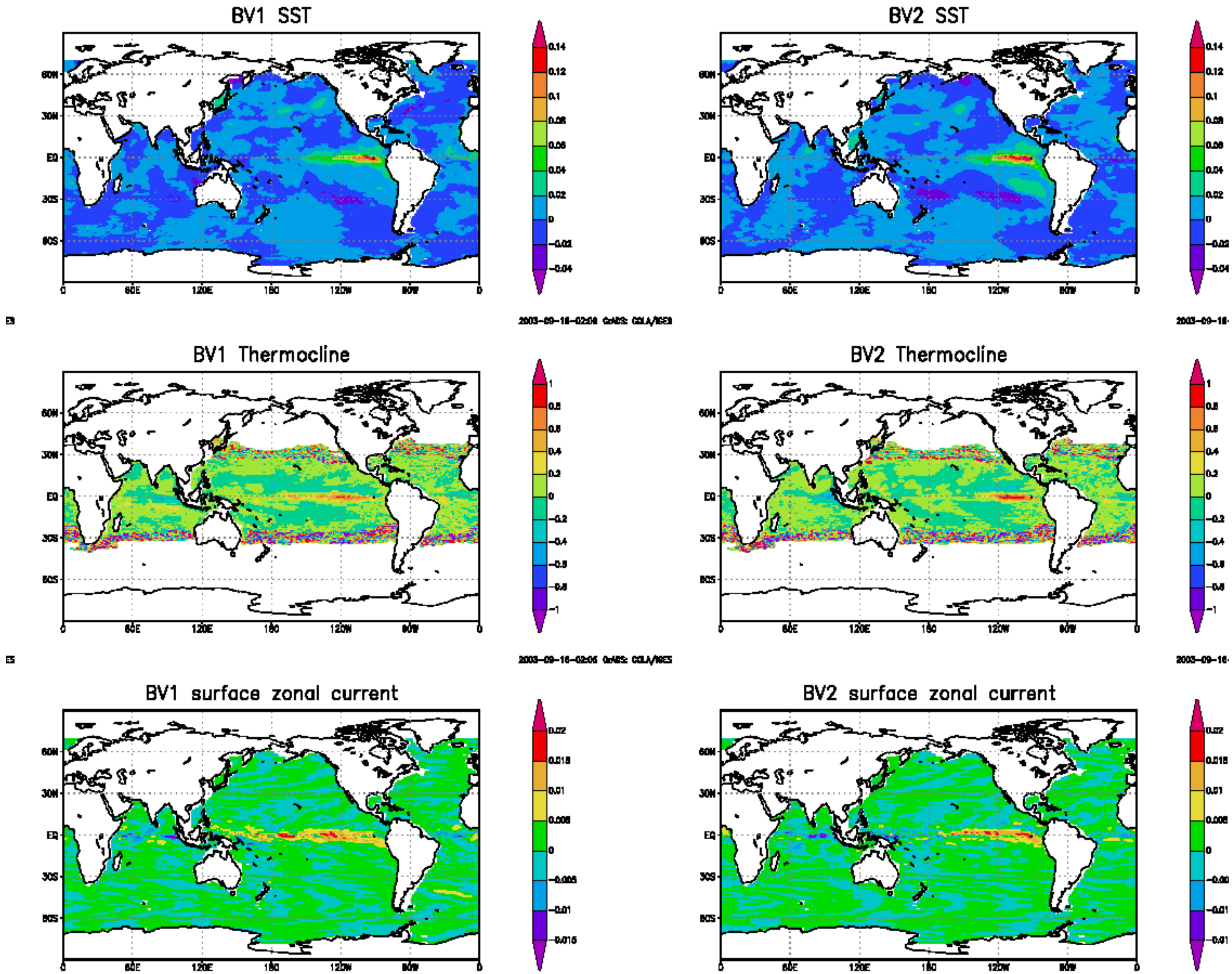
"Spring Barrier": The "dip" in the error growth chart indicates a large error growth for the forecast that begins in the spring and passes through the summer. Removing the projection of the composite BV from the initial conditions (one d.o.f.) wipes it out.

Breeding with the NSIPP coupled GCM

- As in the Lorenz coupled system, we rescale using a slow variable (Nino 3 SST) and an interval long compared to the "weather noise" (one month)
- We performed two independent breeding cycles
- Regressed against their own Nino-3 SST index.
- Performed correlation matrix EOFs, very similar to the regression wrt Nino-3 index
- Results are extremely robust, and almost identical for BV1 and BV2, computed independently.

Regression maps with BV NINO3 index

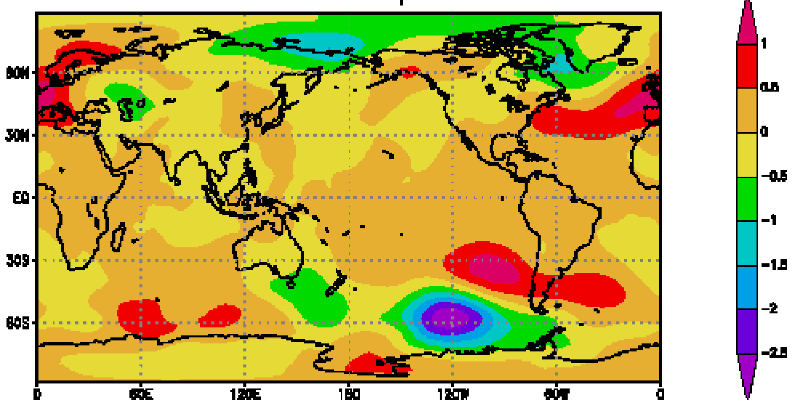
- oceanic variables



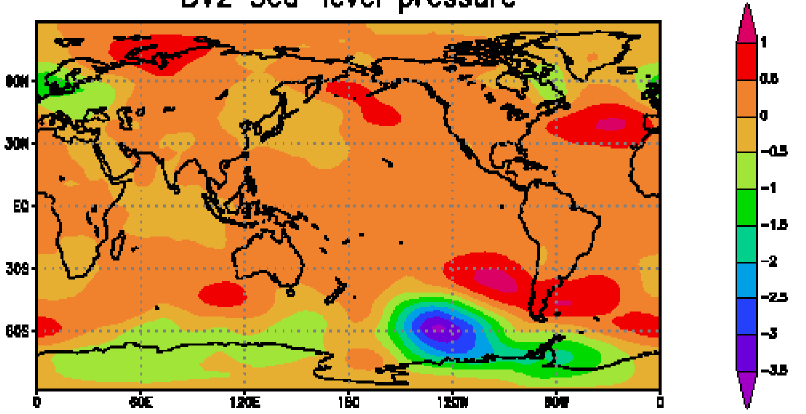
Regression maps with BV NINO3 index

- atmospheric variables

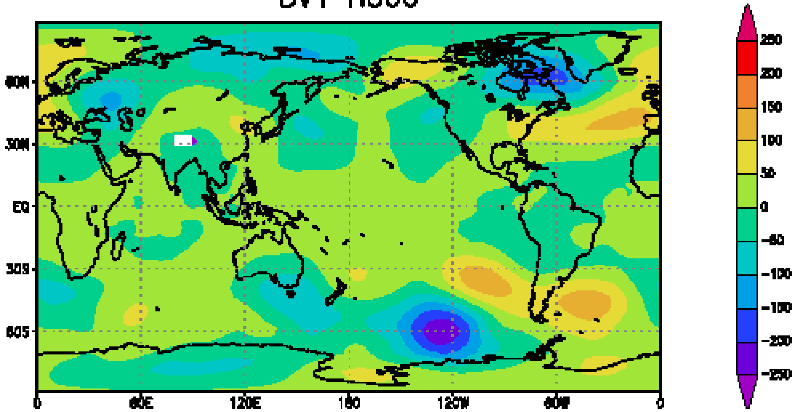
BV1 Sea-level pressure



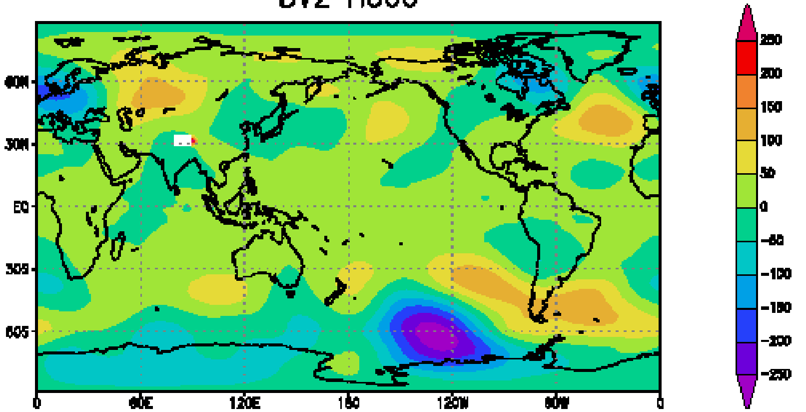
BV2 Sea-level pressure



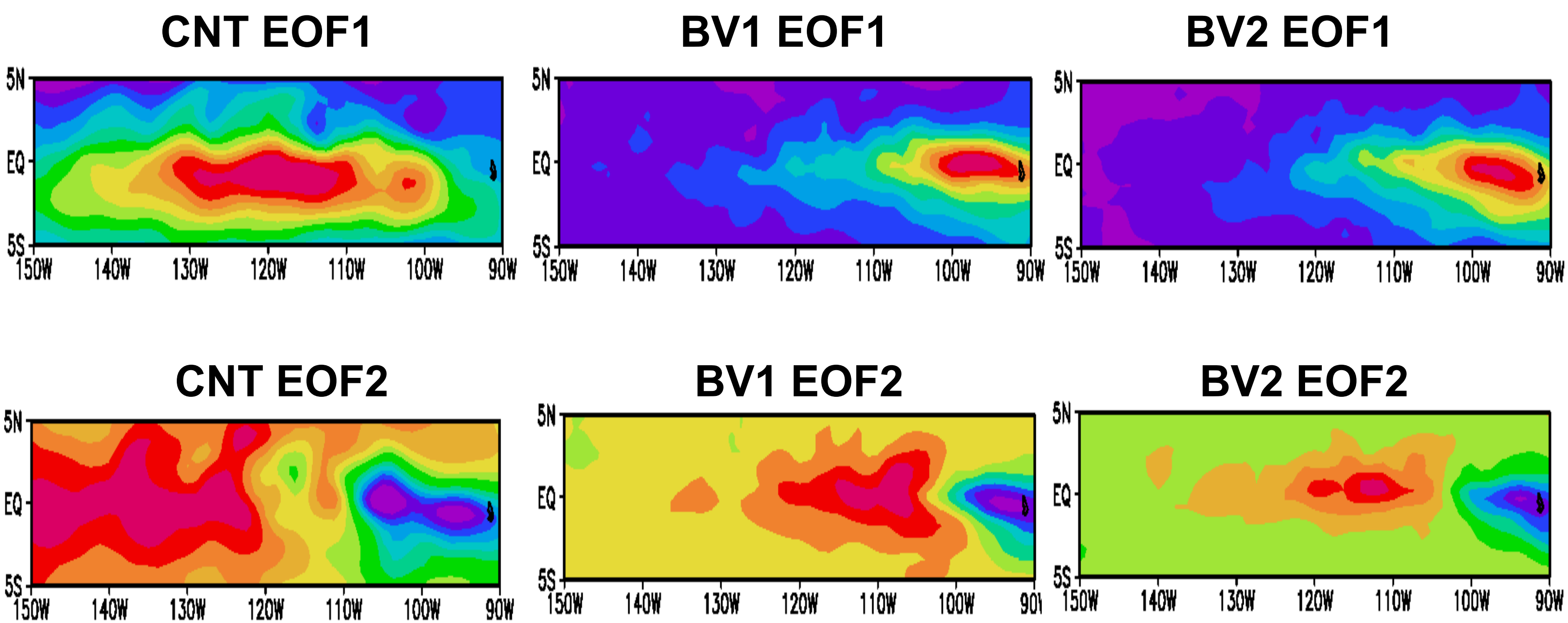
BV1 H500



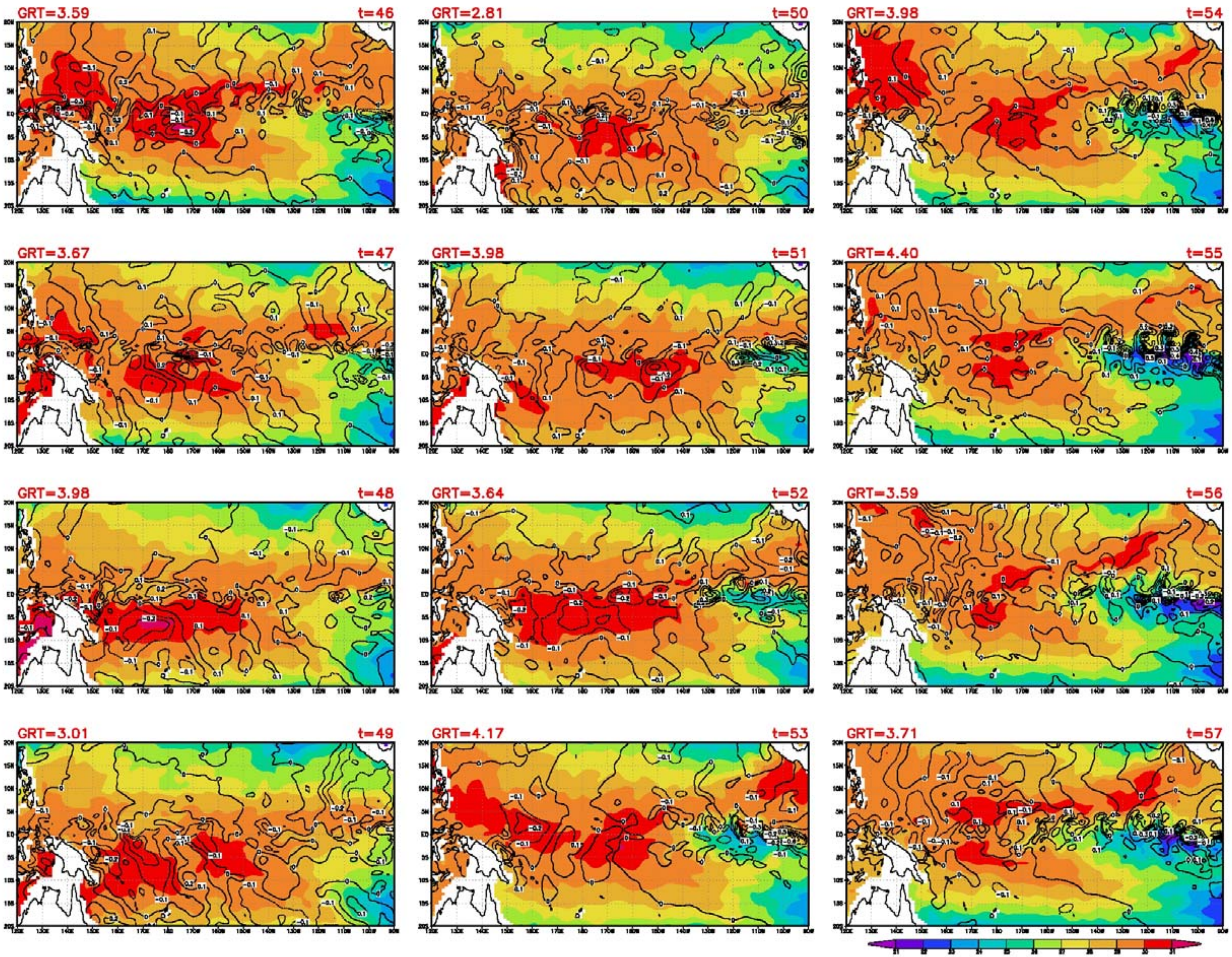
BV2 H500



Background ENSO vs. ENSO "embryo"



Evolution of the control SST (color) and the BV SST (contours) between months t=45 and t=57



Summary about breeding in a coupled system

- Breeding is a simple, finite-time, finite-amplitude generalization of Lyapunov vectors: just run the model twice…
- The only free parameters are the amplitude and frequency of renormalization (does not depend on the norm)
- Breeding on the Lorenz (1963) model yields very robust prediction rules for regime change and duration
- In a coupled models, it is possible to isolate the fast and the slow modes by a physically based choice of the amplitude and frequency of the normalization.

Tentative conclusions about data assimilation in coupled systems with multiple time scales

- In a system with instabilities with multiple time scales, methods that depend on linearization to get the “errors of the day” such as 4D-Var and KF may not work.
- The results using breeding suggest that a coupled Ensemble Kalman Filter could be designed for data assimilation using long time steps

end